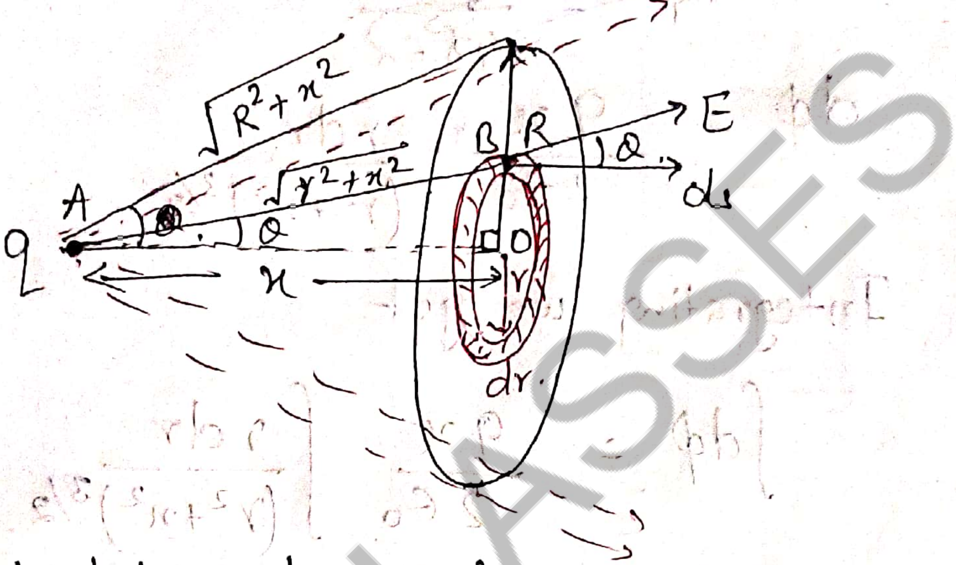
Lecture-15

* Flux due to point Charge through Circular Disk:-

Consider a circular disk of radious "R". A point Charge "q" at x distance from centre of a disk.



Now let disk made up of elementary ring of radious "r" and thickness "dr".

A/fig. Flux $d\phi = \vec{E}\cdot\vec{ds}$

$$d\phi = E\,ds\cos\theta \quad \text{--- (1)}$$

$\Rightarrow \cancel{d\phi}$

$$\left\{ E = K\frac{q}{\left(\sqrt{r^2+x^2}\right)^2} = K\frac{q}{(r^2+x^2)} \right.$$

$$\left. ds = 2\pi r\cdot dr \qquad \cos\theta = \frac{x}{\sqrt{r^2+x^2}} \right\}$$

from eq^n (1)

$$d\phi = K\frac{q}{r^2+x^2}\cdot 2\pi r\,dr\cdot\frac{x}{\sqrt{r^2+x^2}}$$

$$d\phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+x^2)^{3/2}}\, 2\pi r x\, dr$$

$$\cancel{d\phi = \frac{q}{2\epsilon_0} \cdot \frac{r^2 dr}{(r^2+x^2)^{3/2}}}$$

$$\cancel{d\phi = \frac{1}{4\pi\epsilon_0}}$$

$$d\phi = \frac{qx}{2\epsilon_0} \cdot \frac{r\,dr}{(r^2+x^2)^{3/2}}$$

Integrating we get

$$\int d\phi = \frac{qx}{2\epsilon_0} \int \frac{r\,dr}{(r^2+x^2)^{3/2}}$$

$$\phi = \frac{qx}{2\epsilon_0} \int \frac{r\,dr}{(r^2+x^2)^{3/2}}$$

Let $r^2 + x^2 = z^2$

differentiating we get

$$2r\,dr + 0 = 2z\,dz$$

$$r\,dr = z\,dz$$

$$\phi = \frac{qx}{2\epsilon_0} \int \frac{z\,dz}{z^3}$$

$$\phi = \frac{qx}{2\epsilon_0} \int z^{-2} dz$$

$$\phi = \frac{qx}{2\epsilon_0} \left(\frac{-1}{z}\right)$$

$$\phi = -\frac{qx}{2\epsilon_0 Z}$$

Now we put $Z = \sqrt{r^2+x^2}$ then

$$\phi = \left[-\frac{qx}{2\epsilon_0\sqrt{r^2+x^2}}\right]_0^R$$

$$\phi = \left[\frac{-qx}{2\epsilon_0\sqrt{R^2+x^2}} - \left(-\frac{qx}{2\epsilon_0 x}\right)\right]$$

$$\boxed{\phi = \frac{q}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]}$$

$$\boxed{\phi = \frac{q}{2\epsilon_0}(1-\cos\theta)}$$

or $1-\cos\theta = 2\sin^2\theta/2$

$$\phi = \frac{q}{2\epsilon_0} \times 2\sin^2\theta/2.$$

$$\boxed{\phi = \frac{q}{\epsilon_0}\cdot\sin^2\theta/2}$$